Study of variation of G using the cosmological balloon model

Robert Monjo i Agut

Department of Earth Physics and Thermodynamics, Faculty of Physics, University of Valencia. N. 50, Dr. Moliner. 46100 Burjassot (Valencia, Spain)

E-mail: robert@temps.cat

Abstract

Recently, possible variations of 'gravitational constant' G have been measured, which can be explained by the expansion of the universe. Usually cosmological models are built with "dark energy" in order to explain why the expansion of the universe is not slowing by the gravitational action. In this paper we propose an alternative model to explain the variation of *G* with the expansion of the universe, without the need to postulate the existence of dark energy. The model is based on a relativistic adaptation of model of 'cosmological balloon'. The model provides an explanation for the 'gravitational constant' and its variation. In fact, the predicted value of the relative variation ($G^{-1}dG/dt = 7.3 \cdot 10^{-11}$ yr^{-1}) is consistent with most precise observations. Another result of this work is the obtaining of a value for the Hubble parameter equal to 71.3 ± 0.6 $\mathrm{km \cdot s^{-1} \cdot Mpc^{-1}}$, which is consistent with the currently observed value (71 ± 4 $\mathrm{km \cdot s^{-1} \cdot Mpc^{-1}}$). In addition, the model gives theoretical values for the universe energy ($4.10 \cdot 10^{53}$ *kg*), as well as other cosmological quantities, such as the minimum measurable mass ($(1.36 \pm 0.01) \cdot 10^{-67}$ *kg*) and the minimum time interval ($1.4 \cdot 10^{-105}$*sec*).

Keywords: Varying G · Balloon model · Expanding universe

1 Introduction

The observation of all physical quantities, including time and energy, seem to be affected by a quantum phenomenon of delocalization (Weinberg, 1995), so it seems that the cosmological equations must also be submitted to process quantitation (Carlip 2001; Rovelli, 2004). But first, it is necessary to better understand the expansion of the universe and the origin and evolution of the coupling constant of gravitation, *G*.

Dirac (1938) was probably the first to suggest the possibility that *G* might vary depending on the age of the universe, specifically proposed that *G* is a function of inverse time. This theory was developed decades later by Bran and Dicke (1961). Contrary to this, other authors propose that *G* increases with age of the universe (Helling et al. 1983; Abdel-Rahman, 1990; Massa, 1997; Arbad, 2003). At present, many authors conclude that measuring is a variation of *G*, but there is no consensus on the sign nor the magnitude (Salam and Wigner, 1972, Müller et al. 1991; Demarque *et al.* 1994; Garcia-Berro et al. 1995; Thorsett, 1996; Benvenuto *et al.*, 1999; Olive *et al.*, 2002).

Moreover, at present there is a significant advance in the measure of cosmological parameters, and it has obtained results confused for the density of the universe and the expansion. Specifically, a density equal to the critical is found, but with a expansion that is not slowing down (Tegmark et al. 2004; Spergel et al. 2007; Hinshaw et al., 2009). To explain the results, it has been necessary to introduce the concept of dark energy, so that now the expansion is guaranteed by the acceleration caused by such energy, thus compensating to the gravitational attraction of the critical density.

In modern cosmology, four macroscopic dimensions is commonly used, as well as the proposals on the microscopic scale of Kaluza-Klein theory and string theory (Green et al. 1987; Wuensch, 2003). These macroscopic dimensions can be curved by the effect of energy (Einstein, 1916; O'Neill, 1983), so usually it can build models of expansion of the universe by considering the mass-energy that exists. But today, the postulate of the existence of "dark energy" (not observable) has been necessary to explain the fact that the expansion of the universe is not slowing for the gravitational action.

In this paper we propose an alternative model to explain the relationship between the variation of G and the expansion of the universe, without the need to postulate the existence of dark energy. The model is based on a relativistic adaptation of the cosmological ballon model (Eddington, 1933).

The cosmological model of the balloon was used to explain Hubble's law (1937), that is made popular by Eddington (1933) and Hoyle (1960), who explained that, where galaxies drawn on the surface of a balloon that is inflated, these are separated from each other in a way similar to how the universe expand. Therefore, we can make the analogy that the surface of the globe (2D) is the hypersurface (3D) of the universe, so that the radius of the expansion is function of time. The act of inflating the balloon itself is a temporal dimension, but the fact that the balloon should be a curved surface (2D) implies the necessity of it is contained in a volume (3D). Therefore we propose that in the universe happens a similar thing: To better describe the curved hypersurface (3D) of space, we need use a hipervolum (4D), in addition to the temporal dimension independently.

Another possible similarity between the universe and the ballon surface is that the universe is finite (in space and time) and has no privileged points, because its centre is not on the surface. If the universe has a finite age, as observations suggest, and given that the speed of expansion is finite, then it seems impossible that the universe be infinite. This rules out *a priori* that the universe be flat or completely open, because this necessarily implies infinitude, or borders, and therefore a privileged point (centroid). For all this, it seems that only a closed universe, and thus with positive curvature, makes sense relativistic. With these ideas, the simplest model of universe that we have formed is one that has a maximum spatial symmetry: the hypersphere. The cosmological model of balloon must satisfy the general relativity (Einstein 1916). In addition, it is also necessary to postulate some additional relations to explain the ‘gravitational constant’, G.

2 Used assumptions

2.1 Cosmological balloon model

Firstly, we searched a cosmological model that satisfies the relativity, has a positive curvature, and is finite without borders (privileged points). With all this, our model is a universe with 4 dimensions contained in a 5-dimensional space, and with product defined by trace –3; i.e., four spatial dimensions (x, y, z, u), and a temporal (T). Therefore, it is a Lorentz or pseudo-Riemannian variety of signature (1, 4), according to the formal notation (O'Neill, 1983). But, also it is known as 5D Minkowski space (Dvali *et al.*, 2000). Henceforth, we write the vector of space-time as $L = (T, x, y, z, u)$.

The five proposed dimensions are macroscopic unlike that is proposed in theories of Kaluza-Klein (Overduin and Wesson, 1997; Wuensch, 2003). In addition, there must be a linkage so that reduce the configuration space by one topological dimension less (Salvatore and Longoni, 2005). Such condition is given by the following relationship: all the events in the universe $L_i = (T_i, x_i, y_i, z_i, u_i)$ have the same form with respect to a given origin of reference; i.e., there exists a coordinate origin O so that satisfies:

$$\left|L_i - O\right|^2 = \tau_o^{\,2} \quad \forall L_i \tag{2.1}$$

where τ_o is a constant. If we choose the zero origin $O = (0, 0, 0, 0, 0)$, and we suppose time T is much greater than τ_o, then this link condition becomes in the equation of a light hipercon, with focus in the origin O, i.e., for large time the universe seen from the origin takes the form of a 4D hypersurface expanding in function of time T, according to:

$$T^2 \approx x^2 + y^2 + z^2 + u^2 = \vec{r}^{\,2} + u^2 \tag{2.2}$$

where we have defined the vector r as the vector of ordinary spatial coordinates (x, y, z). However, observers are not in the origin, where by definition the time does not pass; but they are part of that expanding universe (hypersurface). Therefore, to choose a new reference system, we need a fixed spatial point but observable, r_o. In other words, the spatial point (r_o) must belong to the universe today, and so must satisfy the Equation 2.2. If we choose zero value for ordinary space components, x, y and z, then necessarily we have that $u = T$, where T is the age of the universe in every moment of the observations. In other words, our spatial reference point is in the path $L = (T, 0, 0, 0, T)$, but in the study of motion, we need fix the reference time in T_o, so finally the "point" of space-time reference is $W \equiv (T_o, 0, 0, 0, T)$.

The reference system TXYZ is constructed with a hyperplane tangent space at the point of reference, and perpendicular to the direction of *U*. The "point" W belongs to this hyperplane and is a no-existent trajectory of space-time, it just satisfies the Equation 2.2 for the reference instant T_o (Figure 1).

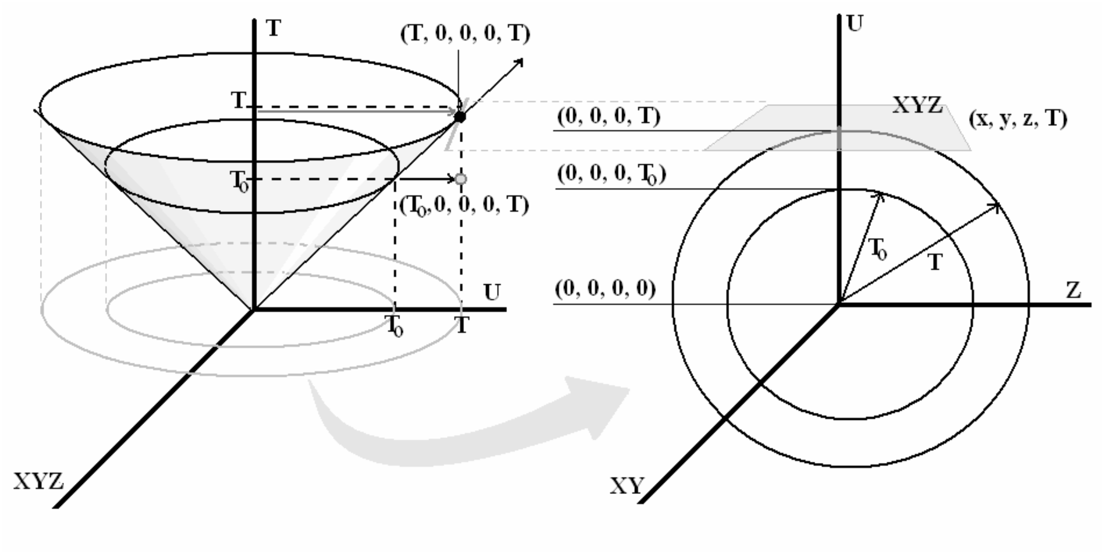

Figure 1. Graphical representation of the 5 dimensions of the universe (left) and projection of the universe on the hyperplane of 4 spatial dimensions (right).

Around the universe next to *W*, the space is almost flat, but if we separate the point of reference, the region belonging to the hypersurface is decreasingly flat. Therefore we define an angle of separation γ that is given by the hypersphera obtained from the Equation 2.2 for each instant *T*. If we define the ordinary distance *r* as the module of the sub-spacial vector (x, y, z), then the sine of the angle of separation is the ratio between *r* and *T*:

$$T^2 = \vec{r}^{\,2} + u^2 \qquad \longrightarrow \qquad \begin{cases} \sin\gamma = \dfrac{r}{T} \\ \cos\gamma = \dfrac{u}{T} \end{cases} \tag{2.3}$$

where the time *T* is considered as the radius. That is, the previous condition of linkage has brought us now to say:

$$u = T\sqrt{1-\frac{r^2}{T^2}} \tag{2.4}$$

In order to describe a motion in the reference system *W*, we can think that a Lorentz transformation is necessary (Møller, 1952). In particular, this coordinate transformation is made from the reference system of *O* to *W*, considering the relative velocity that, in $r = 0$, is $du_w/dT = 1$ (and direction *u*). However, this is only true if the reference system *W* belongs to the universe (according to Equation 2.1 or 2.2), i.e., only for $T = T_o - \tau_o$. Therefore, we must necessarily make a new hypothesis: Our perception of the universe is such that we believe there is a "point" *W*, where we theoretically are; so that any point *s* on the system *W* is written as the difference between the components of *s* and *W*. Put another way, everything happens as if we observe from the point *O*, but the difference between each point *L* and *W*:

$$s \equiv L - W \;=\; (T - T_o,\;\; x,\;\; y,\;\; z,\;\; u - T) \tag{2.5}$$

In other words, in according to our perception, the reference point is *W*. Using Equation 2.4 we can write Equation 2.5 as:

$$s = \left(t,\;\; \vec{r},\;\; -T\left(1-\sqrt{1-{r^2}/{T^2}}\right)\right) \tag{2.6}$$

where we have defined $t \equiv T - T_o$. In addition, if the distance r is small compared to the time T, then we can approximate that:

$$-T\left(1-\sqrt{1-\frac{r^2}{T^2}}\right) \approx -\frac{r^2}{2T} \qquad sii \quad r << T \tag{2.7}$$

Therefore, the four apparent dimensions are explicitly, as:

$$s = (t, \quad \vec{r}, \quad -r^2/2T) \tag{2.8}$$

and then locally (r << T) the space becomes flat and we can reproduce the Minkowski metric. Moreover, since the universe is expanding, we see that for an object that does not change the angle on coordinate system, we have:

$$\sin\gamma \cdot \vec{k}_r = \frac{\vec{r}}{T} \quad \rightarrow \quad 0 = \frac{d\vec{r}}{T} - \frac{\vec{r}}{T^2}dt \quad \rightarrow \quad \frac{d\vec{r}}{dt} = \frac{\vec{r}}{T} \tag{2.9}$$

where $\boldsymbol{k_r}$ is the vector direction of $\boldsymbol{r}$ and γ is the relative angle of the studied object with respect to W. It is advisable therefore to use a spatial variable independent from time, such as the angle. With this, we define comoving spatial coordinates, ***r'***, according to:

$$\vec{r}' \equiv T_o \sin\gamma \cdot \vec{u}_{\vec{r}} \quad \rightarrow \quad \vec{r}' = T_o \frac{\vec{r}}{T} \tag{2.10}$$

Then the Equation 6 can be rewrite as:

$$s = \left(t, \quad \frac{T}{T_o}\vec{r}', \quad -T\left(1-\sqrt{1-\frac{r'^2}{T_o^{\,2}}}\right)\right) \tag{2.11}$$

Moreover, as we have four apparent dimensions in five dimensions, general relativity can be used to reduce them to four spatial dimensions: If there are an observer who is dropped, then their own coordinates $\{\xi^\alpha\}$ can be transformed according to another reference system W $\{x^\varepsilon\}$ using the equivalence principle of Einstein (1916). The square of the module position vector before transformation is:

$$ds^2 = \eta_{\alpha\beta} d\xi^\alpha d\xi^\beta \tag{2.12}$$

where $\eta_{\alpha\beta}$ is the 4-dimensional metric tensor with trace –2, whilst ds is the module of position vector (Einstein, 1916). If $ds^2 > 0$ then the own time $d\tau$ is defined as $d\tau = ds$. And therefore, the change of reference system $\{\xi^\alpha\} \rightarrow W\{x^\varepsilon\}$ is given by:

$$ds^2 = \eta_{\alpha\beta}\frac{d\xi^\alpha}{dx^\lambda}\frac{d\xi^\beta}{dx^\mu}dx^\lambda dx^\mu = g_{\lambda\mu}(x)dx^\lambda dx^\mu \tag{2.13}$$

where the tensor $g_{\lambda\mu}(x)$ is defined as:

$$g_{\lambda\mu} = \eta_{\alpha\beta}\frac{d\xi^\alpha}{dx^\lambda}\frac{d\xi^\beta}{dx^\mu} \tag{2.14}$$

2.2 Energy of the Universe

In our model, the universe has a minimum interval of time, or quantum of time which equals the constant τ_o (Eq. 2.1). Therefore, any spatial variable x^α can be written as an integer number of τ_o according to $x^\alpha = n^\alpha \cdot \tau_o$, where n^α is an integer. However we can assume that, initially, to macroscopic scales such description is almost equivalent to a spacetime continuum, so the differential calculations are still valid.

To understand the discretization of time, we can make an analogy with the energy propagation of a wave $\Psi(T)$ function of time T. In this case the medium where the wave propagates is the 5D space of the universe, and wave is the universe itself (set of possible events that satisfies the linkage condition $L^2 = \tau_o$). The energy (or matter) is propagated by the discrete points, such as those satisfy the condition of quantum ligament, according to:

$$1 = \left(n^0\right)^2 - \left(n^1\right)^2 - \left(n^2\right)^2 - \left(n^3\right)^2 - \left(n^4\right)^2 \tag{2.15}$$

where n^α is the integer associated with the spatial variable x^α. However, due to the rotational symmetry *SO(4)* of points of reference system (Weinberg, 2000), points of space (and therefore matter points) can have "infinite" locations. Therefore we propose a treatment similar to the quantum field theory.

Recall that, whether two magnitudes of a quantum system (Weinberg, 1995, 1996) are represented by observables $\hat{\boldsymbol{A}}$ and $\hat{\boldsymbol{E}}$, which are autoadjunts operators, and are operating on a state space which satisfies the Schwarz inequality, then the expected value of product $\hat{\boldsymbol{A}}\cdot\hat{\boldsymbol{E}}$ is greater than the magnitude of its imaginary part (Robertson, 1929), namely:

$$\left|\langle\psi|\hat{A}\hat{E}|\psi\rangle\right|^2 \geq \left|\frac{1}{2i}\langle\psi|\left[\hat{A},\hat{E}\right]|\psi\rangle\right|^2 \tag{2.16}$$

where $\langle\psi|\boldsymbol{X}|\psi\rangle$ is the expected value of $\boldsymbol{X}$ in state space $|\psi\rangle$, whilst $[\hat{\boldsymbol{A}}\ ,\ \hat{\boldsymbol{E}}] \equiv \hat{\boldsymbol{A}}\hat{\boldsymbol{E}}$ - $\hat{\boldsymbol{A}}\hat{\boldsymbol{E}}$ is the commutator, which also equals $[\hat{\boldsymbol{A}}\ ,\ \hat{\boldsymbol{E}}] = [\hat{\boldsymbol{A}} - \langle\psi|\hat{\boldsymbol{A}}|\psi\rangle,\ \ \hat{\boldsymbol{E}} - \langle\psi\,\hat{\boldsymbol{E}}|\psi\rangle]$, and $\langle\psi|\boldsymbol{X}|\psi\rangle$ is the expected value of $\boldsymbol{X}$ in state space $|\psi\rangle$. Therefore, the standard deviation of the $\hat{\boldsymbol{A}}$ and $\hat{\boldsymbol{E}}$ that satisfy:

$$\Delta\hat{A}\cdot\Delta\hat{E} \geq -\tfrac{1}{2}i\langle\psi|\left[\hat{A},\hat{E}\right]|\psi\rangle \tag{2.17}$$

For example, if we define the momentum-energy operator as $\tilde{p}^{\,\alpha} \equiv i\partial^{\,\alpha}$ acting on any state $|\psi\rangle$, then easy to show that:

$$\Delta\hat{p}^{\alpha}\Delta\hat{x}^{\alpha} \geq \tfrac{1}{2} \tag{2.18}$$

If we do not any measure, then we take the minimum deviations:

$$\Delta\hat{p}^{\alpha}\Delta\hat{x}^{\alpha} = \tfrac{1}{2} \;\;\rightarrow\;\; \Delta\hat{p}^{0}\Delta\hat{x}^{0} = \tfrac{1}{2} \tag{2.19}$$

If we now apply the operators on a hypothetical state of the universe *Ψ(T)* at the minimum interval of time, then the minimum amplitude of momentum-energy at that time interval is:

$$\Delta\hat{p}^{0}(\Delta x^{0}{}_{\min}) = \frac{1}{2\Delta x^{0}{}_{\min}} = \frac{1}{2\tau_o} \tag{2.20}$$

By definition, this is the total mass-energy of the universe (M_U), which spreads from the moment τ_o. In units of the International System, we have:

$$M_U \equiv \Delta\hat{p}^{0}(\Delta x^{0}{}_{\min}) = \frac{\hbar}{2\tau_o} \tag{2.21}$$

In general, the maximum deviation of a spatial component is $\Delta x^0{}_{\max} = \Delta n^0\tau_o = T = n^0\tau_o$. Therefore, the minimum deviation associated with this, $\Delta p^0{}_{\min}$, is:

$$\Delta\hat{p}^{0}(\Delta x^{0}{}_{\max}) = \frac{1}{2\Delta x^{0}} = \frac{1}{2\tau_o n^0} = \frac{M_U}{n^0} \equiv \tilde{m}_o(n^0) \tag{2.22}$$

where $\tilde{m}_o$ is the minimum measurable energy in the universe, and therefore the quantum of spacetime-energy.

2.3 Density of the Universe

Under the proposed model, the volume (3D) of the universe is $2\pi^2 T_o^{\,3}$, therefore the average energy density of the universe is:

$$\rho_o \equiv \frac{M_U}{\Omega T_o^{\,3}} \tag{2.23}$$

where $\Omega \equiv 2\pi^2$, and M_U is the total mass of the universe. Assuming that the density is distributed uniformly in the universe, then we can define a mass M_γ closed in a volume of radius equal to the arc $D = \gamma/T_o$, as:

$$M_\gamma(D) \equiv \rho_o \omega_\gamma D^3 \tag{2.24}$$

where $\omega_\gamma \equiv 2\pi(\gamma - \sin\gamma \cdot \cos\gamma)\cdot\gamma^{-3}$ is the volumetric angle associated with the angle γ, with respect to a given reference system (see Appendix A)

Moreover, the linear density μ of energy in the universe, M_U, is defined as the ratio between energy and age of the universe T_o, namely:

$$\mu \equiv \frac{M_U}{T_o} = \frac{\tilde{m}_o}{\tau_o} \tag{2.25}$$

With this we can rewrite Equation 2.23 as:

$$\rho_o = \frac{\mu}{\Omega T_o^{\,2}} \tag{2.26}$$

2.4 Approach of heterogeneous universe

Assuming for a particular reason, in a moment T_o, a number of points n_M of energy $\tilde{m}_o$ manage to form an accumulation of energy M = $n_M\tau_o$, then we can define a time built by the total length of all points, i.e., $T_M = n_M\tau_o$. Thus, the linear density is the same as in the universe in T_o, i.e. μ. In fact, using Equation 2.25 and 2.22, we see that:

$$\mu = \frac{M_U}{T_o} = \frac{n_o\tilde{m}_o}{n_o\tau_o} = \frac{n_M\tilde{m}_o}{n_M\tau_o} = \frac{M}{T_M} \tag{2.27}$$

Then the time T_M represents the radius of curvature of a hypothetical micro-universe with the same linear density μ than the universe, and with a mass-energy M is distributed uniformly in the space of this micro-universe, i.e., with constant density ρ_M, and greater than the universal density, ρ_o. However the density of the accumulation is not always constant in space.

Whether the accumulation is confined or not in a sphere of radius R, for any spatial arc D we can define an average density of that accumulation ρ_D, which in general is no constant (different of ρ_M). Therefore, such density is given by the ratio between the energy, $m(D) \leq M$, closed by the symmetric volume of the arc D, and this volume, that is:

$$\rho_D(D) \equiv \frac{m(D)}{\omega_\gamma D^3} \tag{2.28}$$

Recall that a relationship exists between the curvature of the universe, T_o, and the average density, ρ_o, which is given by Equation 2.26, and therefore:

$$T_o = \sqrt{\frac{\mu}{\Omega\rho_o}} \tag{2.29}$$

Therefore, we the make assumption that, in order to study the behavior of the accumulation of points, we can have a radius of curvature T_r associated with the average density $\rho_D(D)$, so that in the limit when $\rho_D(D) \to \rho_o$ we recover $T_r \to T_o$, therefore necessarily we define:

$$T_r \equiv \sqrt{\frac{\mu}{\Omega\rho_D}} \tag{2.30}$$

We see that, if the average density of the accumulation ρ_D is constant with distance ($\rho_D(D) = \rho_M$), then we obtain that T_r curvature is also constant and equal to T_M. In order to show this, we applied that the density (constant) of the accumulation satisfies the Equation 2.23, with energy M of the micro-universe and hence we find that:

$$If \quad \rho_D(D) = \rho_M \equiv \frac{M}{\Omega T_M^{\,3}} \qquad \longrightarrow \qquad T_r = \sqrt{\frac{\Omega T_M^{\,3}\mu}{\Omega M}} = T_M \tag{2.31}$$

3 Results

3.1 Obtaining the differential of space-time

From Equation 2.11, we can calculate the line element of the universe with respect to our reference frame of *W*; and therefore, this is equivalent to the differential line with respect to any rest point (expanding). According to Equation 2.11, the line *ds* can be written as:

$$ds = \left(dt,\ \ \frac{T}{T_o} d\vec{r}\,' + \frac{dt}{T_o}\vec{r}\,',\ \ -dt\left(1-\sqrt{1-\frac{r'^2}{T_o^{\,2}}}\right) + \frac{T}{T_o} dr'\left(\frac{r'}{T_o}\right)\left(1-\frac{r'^2}{T_o^{\,2}}\right)^{-\frac{1}{2}} \right) \tag{3.1}$$

where *r'* are the comoving coordinates in space XYZ (Eq. 2.10). And therefore, the square of the differential is:

$$\begin{aligned} ds^2 = dt^2 - \left(\frac{T}{T_o}\right)^2 d\vec{r}\,'^2 \ - \left(\frac{\vec{r}\,'}{T_o}\right)^2 dt^2 - 2\left(\frac{T}{T_o}\right)\left(\frac{\vec{r}\,'}{T_o}\right) dt d\vec{r}\,' \ - \left(1-\sqrt{1-\frac{r'^2}{T_o^{\,2}}}\right)^2 dt^2 - \\ - \left(\frac{T}{T_o}\right)^2 \frac{\left(\frac{r'}{T_o}\right)^2}{1-\frac{r'^2}{T_o^{\,2}}} dr'^2 \ - \ 2\left(1-\sqrt{1-\frac{r'^2}{T_o^{\,2}}}\right)\left(\frac{T}{T_o}\right)\frac{\frac{r'}{T_o}}{\sqrt{1-\frac{r'^2}{T_o^{\,2}}}} dt dr' \end{aligned} \tag{3.2}$$

operating the square of the fifth term on the right of equality:

$$\begin{aligned} ds^2 = dt^2 - \left(\frac{T}{T_o}\right)^2 d\vec{r}\,'^2 \ - \left(\frac{\vec{r}\,'}{T_o}\right)^2 dt^2 - 2\left(\frac{T}{T_o}\right)\left(\frac{1}{T_o}\right) dt \overbrace{\vec{r}\,' d\vec{r}\,'}^{\substack{r'dr' \\ \uparrow}} - \left(-\frac{r'^2}{T_o^{\,2}} + 2\left(1-\sqrt{1-\frac{r'^2}{T_o^{\,2}}}\right)\right) dt^2 \\ - \left(\frac{T}{T_o}\right)^2 \frac{\left(\frac{r'}{T_o}\right)^2}{1-\frac{r'^2}{T_o^{\,2}}} dr'^2 \ - \ 2\left(\frac{T}{T_o}\right)\frac{r'}{T_o}\left(\frac{1}{\sqrt{1-\frac{r'^2}{T_o^{\,2}}}} - 1\right) dt dr' \end{aligned} \tag{3.3}$$

and finally, by grouping terms:

$$ds^2 = dt^2\left(2\sqrt{1-\frac{r'^2}{T_o^{\,2}}} - 1\right) \ - \ \left(\frac{T}{T_o}\right)^2 \left[\frac{dr'^2}{1-\frac{r'^2}{T_o^{\,2}}} + r'^2\, d\Omega^2\right] \ - \ 2\left(\frac{T}{T_o}\right)\left(\frac{r'}{T_o}\right)\frac{drdt}{\sqrt{1-\frac{r'^2}{T_o^{\,2}}}} \tag{3.4}$$

For the movements in that do not changes the angle, $d\Omega = 0$, then we can write that:

$$ds^2 = dt^2\left(2\sqrt{1-\frac{r'^2}{T_o^{\,2}}} - 1\right) \ - \ \left(\frac{T}{T_o}\right)^2 \frac{dx_i' dx_i'}{1-\frac{r'^2}{T_o^{\,2}}} \ - \ 2\left(\frac{Tx_i}{T_o^{\,2}}\right)\frac{dt \cdot dx_i'}{\sqrt{1-\frac{r'^2}{T_o^{\,2}}}} \tag{3.5}$$

This is the square of the differential line for small regions of the universe. Then we see that the elements of the tensor *g*, given by Equation 2.13 and 2.14 are:

$$g_{00} = \left(2\sqrt{1-\frac{r'^2}{T_o^{\,2}}} - 1 \right), \quad g_{ii} = -\left(\frac{T}{T_o}\right)^2 \frac{1}{1-\frac{r'^2}{T_o^{\,2}}}, \quad g_{0i} = -\left(\frac{T}{T_o}\right)\left(\frac{x_i'}{T_o}\right)\frac{1}{\sqrt{1-\frac{r'^2}{T_o^{\,2}}}} \tag{3.6}$$

Moreover, if the distance is much smaller than the time (Eq. 2.7) then, according to the Taylor development, we can approximate to:

$$g_{00} \approx \left(1-\left(\frac{\vec{r}'}{T_o}\right)^2 \right), \quad g_{ii} \approx -\left(\frac{T}{T_o}\right)^2 \left(1+\left(\frac{x_i'}{T_o}\right)^2 \right), \quad g_{0i} \approx -\left(\frac{T}{T_o}\right)\left(\frac{x_i'}{T_o}\right) \tag{3.7}$$

Taking the Equations 3.6, the spatial differential dl is (Wald, 1984):

$$d\ell^2 = \left(g_{ii} - \frac{g_{0i}^{\;2}}{g_{00}} \right) dx_i^{\,2} = -\left(\frac{T}{T_o}\right)^2 \frac{1}{1-\left(\frac{r'}{T_o}\right)^2} \left(1 + \frac{\left(\frac{x_i'}{T_o}\right)^2}{\left(2\sqrt{1-\frac{r'^2}{T_o^{\,2}}} - 1 \right)} \right) dx_i^{\,2} \tag{3.8}$$

and again, using development of Taylor:

$$d\ell^2 \approx -\left(\frac{T}{T_o}\right)^2 \left[1 + 2\left(\frac{r'}{T_o}\right)^2 \right] dx_i^{\,2} \tag{3.9}$$

These are the equations of an inflationary universe with a constant increasing is a function of time of the universe, T. Also it is observed that the universe has positive curvature according to Robertson-Walker metric (Wald, 1984), and locally it is flat ($k \approx 2/T_o^2 \approx 0$).

$$ds^2 \approx dt^2 - \left(\frac{T}{T_o}\right)^2 \left(\frac{dr'^2}{1-kr'^2} + r'^2\, d\Omega^2 \right) \;\rightarrow\; k \approx \frac{2}{T_o^{\,2}} \tag{3.10}$$

3.2 Obtaining of varying G

With the approach of the metric tensor of the Equation 3.7 and the change of variables between the distance *r'* and the arc *D*, we can write that:

$$g_{00} \approx 1-\left(\frac{r'}{T_o}\right)^2 = 1-\left(\frac{D}{T_o}\right)^2 \frac{T_o^{\,2}\sin^2\gamma}{T_o^{\,2}\gamma^2} = 1-\left(\frac{D}{T_o}\right)^3 \frac{T_o}{D}\frac{\sin^2\gamma}{\gamma^2} \tag{3.11}$$

Given the definitions for the Equations 2.23 and 2.24, we obtained:

$$g_{00} \approx 1-\frac{\Omega}{\omega_\gamma}\frac{M_\gamma(D)}{M_U}\frac{T_o}{D}\frac{\sin^2\gamma}{\gamma^2} \tag{3.12}$$

Finally, with the definition of linear density (Eq. 2.25), we can write:

$$g_{00} \approx 1-\frac{M_\gamma(D)}{D}\left(\frac{\Omega\sin^2\gamma}{\omega_\gamma\gamma^2\mu} \right) \tag{3.13}$$

where *r'* is the module of the ordinary spatial vector (*x'*, *y'*, *z'*), comoving with universe. Recall that the metric element g_{oo} provides information on the effectiveness of the module of time *T*, when it is an inner product (Wald, 1984), then we can understand as a kind of "density of time" in a certain region of space. Thus, we see that the linear density $\mu_\gamma$behaves as an normalizing factor of this "density of time".

From Equation 3.13 shows that as observed with a larger arc *D*, the element g_{oo} increases with the square of *D*, although the energy closed by *D* increases with the cube.

Moreover, if we consider the approximation of heterogeneous universe we can see that generally the self curvature of the accumulation will follow the same mathematical development that the whole universe (from 3.1 to 3.7). Therefore, considering the Equations 2.30 and 2.28, we obtain that:

$$g_{00} \approx 1-\left(\frac{r'}{T_r}\right)^2 = 1-\frac{\rho_D \Omega D^2}{\mu}\frac{\sin^2\gamma}{\gamma^2} = 1-\frac{m(D)}{D}\left(\frac{\Omega \sin^2\gamma}{\omega_\gamma \gamma^2 \mu}\right) \tag{3.14}$$

where r' is the radial position comoving with respect to the accumulation center. Consequently, there is a difference from Equation 3.13, and it is that now, the element g_{oo} refers to the *time density* of the accumulation in particular and not of the universe in general. And since the density ρ_r is greater than ρ_o, then so is the associated curvature (the radius T_r is less than T_o).

From now onwards, for convenience we define the gravitational varying $G_\gamma = G_\gamma(\gamma, \mu)$ as:

$$G_\gamma \equiv \frac{\Omega \sin^2\gamma}{2\omega_\gamma \gamma^2 \mu} \tag{3.15}$$

where $\Omega = 2\pi^2$, $\omega_\gamma = 2\pi(\gamma - \sin\gamma \cdot \cos\gamma)\cdot\gamma^{-3}$ and γ is the angle in the component u of a point on the accumulation center. Thus, the Equation 3.14 is as:

$$g_{00} \approx 1-\frac{2G_\gamma m(D)}{D} \tag{3.16}$$

where we measure locally (for $\gamma = 0$), G_γ is the "Newton's gravitational constant" (Weinberg, 1972; Wald, 1984), which specifically applies as (from Eq. 3.15):

$$G_o = \frac{3\pi}{4\mu_o} \tag{3.17}$$

Thus, the gravitational varying G_γ can be written according to a funtion of G_o, as:

$$G_\gamma = \frac{4\gamma \sin^2\gamma}{6(\gamma - \sin\gamma \cdot \cos\gamma)} G_o \approx \left(1-\frac{\gamma^2}{7.5}\right) G_o \tag{3.18}$$

In Equation 3.17 we observe that the 'gravitational constant' G_o depends on the linear density of energy, μ_o, and this is not a constant but decreases with time, or put another way, G_o increases. Knowing that the age of the universe is $T_o = (1.373 \pm 0.012)\cdot 10^{10}$ anys (Hinshaw *et al.*, 2009), we can estimate the relative change of G_o, which is given by:

$$G_o = \frac{3\pi}{4}\frac{T_o}{M_U} \qquad \longrightarrow \qquad \frac{1}{G_o}\frac{\partial G_o}{\partial T_o} = \frac{1}{T_o} \approx 7.3 10^{-11}\, year^{-1} \tag{3.19}$$

Moreover, for comoving distances r' of the same order that T_r, the approximation 3.14 is not valid, but it must use the equation 3.6. Therefore, the element g_{oo} the metric tensor is:

$$g_{00} = \left(2\sqrt{1-\frac{r'^2}{T_r^2}} - 1\right) = \left(2\sqrt{1-\frac{2G_\gamma m(D)}{D}} - 1\right) \tag{3.20}$$

4 Discussion

4.1 Expansion of the universe

In cosmology, Hubble's law relates the velocity of an object comoving with universe and its distance from an observer using a slope called Hubble parameter, H_o (Liddle, 2003). The model proposed in this paper describes a universe that has a linear expansion with time (Equation 2.2).

In fact, from the Equation 2.9 and later it is inferred that the expansion is proportional to the inverse of the age of the universe. Therefore, in this case we have obtained that the Hubble parameter is $H_o = 1/T_o$. Knowing that the age of the universe is $T_o = (1.373 \pm 0.012)\cdot 10^{10}$ years (Hinshaw *et al.* 2009) then it follows that the Hubble parameter is 71.3 ± 0.6 km·s^{-1}·Mpc^{-1} which is consistent with observations recently measured values around 71 ± 4 km·s^{-1}·Mpc^{-1} (Spergel *et al.*, 2003; Tegmark *et al.*, 2004) and 70 ± 3 km·s^{-1}·Mpc^{-1} (Spergel *et al.*, 2007).

These results imply that the expansion rate is equal to the light velocity (or very close), as opposed to proposing other authors, who speak of an accelerated expansion (Riess *et al.* 1998; Tegmark *et al.*, 2004; Szabó *et al.* 2007; Kowalski *et al.*, 2008, Komatsu *et al.*, 2009). However, these works using 'luminous distances' sometimes are over the age of the universe, which is impossible to suppose that the speed of light has been constant since the information can not travel more beyond the speed of light (Wald, 1984, and Appendix B). It is possible that the calculation of distances has been made taking into account a possible variation of the 'gravitational constant' (Gaztañaga *et al.*, 2001).

Moreover, our proposed model describes a positive curvature (equation 3.10) which is locally aproximable to zero, ie, the universe is locally plane, which is compatible with observations (Spergel *et al.*, 2007). Specifically, we found that it locally is:

$$k \approx \frac{2}{T_o^{\ 2}} \approx 1.07 \cdot 10^{-20}\, year^{-2} \tag{4.1}$$

where k is the local curvature and T_o is the age of the universe.

In oter hand, according to observations of the cosmic infrared background dipole (Smoot, 1992), there appears to be an absolute reference frame at rest (Cahill and Kitto, 2003; Múnera, 2009). Therefore, we can say that the Earth is moving at about 370 ± 20 km/s with respect to the background radiation. That is, we can build a reference system with rest axes (according to background radiation). This 4D coordinate system is perfectly compatible with our cosmological balloon model. The background radiation in some ways represents the surface of the expanding balloon; and the balloon's radial direction (time) represents the direction of the trajectories of the points with rest. Thus, our galaxy is moving with a "tangential component" with respect to these trajectories with rest.

4.2 Density and mass of universe

The 'gravitational constant', G_o, was first introduced by Newton in 1687, and today it is known empirically that the value is approximately $6.674 \cdot 10^{-11}$ (Mohr *et al.* 2005; Fixler *et al.* 2007). This value allows us to estimate the total energy of the universe and the average density, according to Equations 2.26, 3.17 and 2.23, with $\Omega = 2\pi^2$:

$$\rho_o = \frac{3}{8\pi G_o T_o^{\ 2}} = \frac{M_U}{\Omega T_o^{\ 3}} \tag{4.2}$$

Given the natural transformation of coordinates ($c = 1 = \hbar$) in the international system, and taking the empirical value of the age of the universe as $13.7 \cdot 10^9$ years, we get that:

$$M_U = \frac{3\Omega}{8\pi G_o} T_o = \frac{6\pi^2 c^3}{8\pi \cdot 6{,}67 \cdot 10^{-11}} (\tfrac{kg}{s})(13{,}7 \cdot 10^9\, y)(365\, \tfrac{d}{y})(24\, \tfrac{h}{d})(3600\, \tfrac{s}{h}) = 4{,}10 \cdot 10^{53}\, kg \tag{4.3}$$

Furthermore, from Equation 4.2 we obtain that the density of the universe is:

$$\rho_o = \frac{3}{8\pi G_o T_o^{\ 2}} = 9{,}60 \cdot 10^{-27}\, kg \cdot m^{-3} \tag{4.4}$$

This corresponds to the average density in the literature called critical density, $\rho_c = 9.4 \pm 0{,}8 \cdot 10^{-7}$ $kg \cdot m^{-3}$ (Tegmark, 2004; Spergel, 2007), in fact, observed value of the average density of the universe (relative to the critical density) is 1.0050 ± 0.0060 (Hinshaw *et al.*, 2009), and so there is good agreement between theory and observations.

4.3 Variation of the 'gravitational constant'

In this work we have seen that we expect a relative change of G_o, about $7.3 \cdot 10^{-11}$. Then, this value is consistent with experimental observations: it was measured as the relative variation of G is probably between 10^{-10} and 10^{-10} year^{-1} (Table 1), but sign depends mainly on whether it is measured in a fixed area (e.g. in the solar system) or when measured for different distances of the universe (such as pulsars, white dwarfs, etc.). In the first case, the relative variation of G is positive (Reasenberg and Shapiro 1978, Williams et al. 1996; Biskupek and Müller, 2007), and in the second case seems to be mainly negative (Kasper *et al.* 1994; García-Berro, 1995; Bisnovatyi-Kogan, 2006), but these latest measures are generally less accurate.

Taula 1. Relative variation of G, measured locally (solar system) and cosmologically. Top shows the studies that measure an increase and bottom shows the measuring an decrease.

$G^{-1}dG/dt$ (year^{-1})	Authors	Methodology
$(2 \pm 7) \cdot 10^{-12}$	Müller and Biskupek (2007)	Lunar laser ranging
$\leq 1.6 \cdot 10^{-12}$	Guenther *et al.* (1998)	Helioseismology
$\sim 8 \cdot 10^{-12}$	Williams *et al.* (1996)	Lunar laser ranging
$\sim 10^{-11}$	Krauss and White (1992)	Gravitational Lensing
$\sim 10^{-11}$	Sisterna and Vucetich (1991, 1994)	Palaeontological evidences
$(0 \pm 2) \cdot 10^{-12}$	Anderson *et al.* (1991)	Planetary radar ranging
$(0.2 \pm 0.4) \cdot 10^{-11}$	Hellings *et al.* (1989)	Planetary radar ranging
$(1.6 \pm 0.6) \cdot 10^{-11}$	Van Flandern (1981)*	Lunar laser ranging
$(2 \pm 4) \cdot 10^{-12}$	Hellings *et al.* (1983)	Solar evolution
$\sim 1.5 \cdot 10^{-10}$	Anderson *et al.* (1978)	Planetary radar ranging
$(6.2 \pm 3.3) \cdot 10^{-10}$	Reasenberg and Shapiro (1978)	Planetary radar ranging
$\sim 3 \cdot 10^{-11}$	Williams *et al.* (1978)	Lunar laser ranging
$\leq 1 \cdot 10^{-10}$	Chin and Stothers (1976)	Solar evolution
$(5 \pm 1)\ 10^{-11}$	Dearborn and Schramm (1974)	Stability of galaxy clusters
$\sim 4 \cdot 10^{-11}$	Morrison (1973)	Lunar evolution by using elipses
$\sim 4 \cdot 10^{-10}$	Shapiro *et al.* (1971)	Planetary radar ranging
$(-0.6 \pm 2) \cdot 10^{-12}$	Bisnovatyi-Kogan (2006)	*Pulsar system*
$\geq -4{,}1 \cdot 10^{-10}$	Biesiada and Malec (2004)	*Pulsar system*
$\geq -1 \cdot 10^{-11}$	Gaztañaga *et al.* (2001)	Luminosity of Supernova Ia
$-(1.4 \pm 2.1) \cdot 10^{-11}$	Degl'Innocenti *et al.* (1996)	*Globular clusters*
$-(0.6 \pm 4.2) \cdot 10^{-12}$	Thorsett (1996)	Mass of neutron stars
$-(1 \pm 1) \cdot 10^{-11}$	García-Berro (1995)	white dwarfs, (C/O) stratified
$-(3 \pm 3) \cdot 10^{-11}$	García-Berro (1995)	white dwarfs, non-stratified
$-(9 + 18) \cdot 10^{-12}$	Kaspi *et al.* (1994)	*Pulsar system*
$-(1.1 \pm 1.1) \cdot 10^{-11}$	Damour i Gundlach, (1991)	*Pulsar system*
$\geq -1 \cdot 10^{-12}$	Wang (1991)	Solar luminosity
$\geq -8 \cdot 10^{-12}$	McElhinny *et al.* (1978)	Planets and Dirac creation

The string theory is compatible with negative values, specifically $G^{-1}dG/dt = -1 \cdot 10^{-11 \pm 1}$ year^{-1} (Wu and Wang, 1986), which involve a creation energy (Dirac, 1936 1975). This is that some authors assume as McElhinny *et al.* (1978) and Van Flandern (1981) to explain the planetary acceleration, avoiding taking the change as positive $G^{-1}dG/dt$.

In fact, most works which describe a negative variation of $G^{-1}dG/dt$ assumed this *a priori* (based on Dirac, 1938) and therefore in some cases are looking for an "upper limit" for negative variation, leaving aside the possibility that the value be positive. Also, keep in mind that measures taken for each time a larger cosmological scale, the parameter G_γ is becoming smaller (equation 3.18), which can make it appear a relative decrease of G_γ.

According to Table 1, the best estimates of the relative variation of G are: $(6.2 \pm 3.3) \cdot 10^{-10}$ year^{-1} (Reasenberg and Shapiro, 1978), $(5 \pm 1) \cdot 10^{-11}$ year^{-1} (Dearborn and Schramm, 1974), and (1.6 ± 0,.6) $\cdot\ 10^{-11}$ year^{-1} (Van Flandern, 1981) in the case (*) to suppose that there is no creation of Dirac. That is, it seems more likely that the relative variation of G to be positive than negative.

For all these reasons, for to make the average observed values, the negative measures were discarded. Thus, we obtain the average observed a 95% confidence interval is $G^{-1}dG/dt = (6 \pm 5) \cdot 10^{-11}$ year^{-1}. This value shows that the model is consistent with observations.

4.4 Minimum time and energy

Also, remember that for the initial hypothesis, there is a minimum interval of time which is related to the mass of the universe using a quantum way, according to Equation 2.21, as:

$$M_U \tau_o = \tfrac{1}{2} \tag{4.5}$$

Moreover, given the Equation 3.17, and making the conversion from natural units ($\hbar = 1 = c$) to the International System of Units (SI), we obtain that:

$$\tau_o = \frac{4\hbar G_o}{6\pi T_o c^2} = 1{,}4 \cdot 10^{-105}\, segons = 4{,}3 \cdot 10^{-97}\, metres \tag{4.6}$$

where the error interval is 10%. In addition, according to the Equation 2.22 we obtain the minimum energy of the universe $\tilde{m}_o$, i.e.:

$$\tilde{m}_o T_o = \tfrac{1}{2} \tag{4.7}$$

making the conversion from to the International System of Units, we obtain:

$$\tilde{m}_o(T_o) = \frac{\hbar}{2T_o} = (1{,}36 \pm 0{,}01) \cdot 10^{-67}\, kg \tag{4.8}$$

Therefore, this is the minimum measurable mass and therefore can be considered the rest mass of the space-time, or photon. We can see that this value is far greater than the limit observed for the photon, which is $<10^{-52}$ kg (Eidelman *et al.*, 2004).

4.5 The metric singularity

In the Equation 3.20, we see certain the element g_{oo} can be zero by certain conditions. If the total energy M is closed within a radius $R \leq D$ sufficiently small, then the energy at the distance D also be $m(D) = M$, and the element g_{oo} becomes zero at a certain distance D_o, that is no the radius of Schwarzschild (1916 and 1916b) which is $r_M \equiv 2G_o M$, but this with a factor of 4/3:

$$g_{00} = 0 \quad \leftrightarrow \quad 1 - \frac{r_M}{D_o} = \frac{1}{4} \qquad \longrightarrow \qquad D_o = \frac{4}{3} r_M \tag{4.9}$$

Note that the element g_{oo} presents possibles values for an interval of distances between $4r_M/3$ and r_M, where this values have a negative sign (density of negative time). The interpretation of this result can give rise to various speculations about travel back in time, but we must consider that the Schwarzschild radius is a physical barrier very difficult to overcome (because the density of time is zero; see Eq. 3.13).

5 Conclussions

The universe can be described with 5 dimensions and a condition of linkage which involves an additional connection between space and time. Everything happens as if our observations set artificially an reference instant, and therefore it would be a point that does not exist in the universe. All this implies that the expansion and over time are similar phenomena, since the expansion rate is the same as the speed of time, light velocity ($c \equiv 1$). Therefore, the Hubble parameter equals the inverse of the age of the universe, $H_o = 1/T_o = 71.3 \pm 0.6$ km$\cdot$s$^{-1}\cdot$Mpc^{-1},, which is consistent with more recent observations.

With the additional hypothesis of the existence of a minimum time interval (time quantum, equal to $1.4 \cdot 10^{-105}$s), we have estimated a theoretical value for energy in the universe $M_U = 4.10 \cdot 10^{53}$ kg which determines an average value of energy density $\rho_o = \rho_c = 9.60 \cdot 10^{-27}$ kg$\cdot$m^{-3}, consistent with the observed relative value (1.005 ± 0006 times the critical density, ρ_c).

In addition, this model explains that the "gravitational constant" G is given by the inverse of the linear density of energy of the universe, and therefore, predicts that is dependent on the age of the universe. Specifically, the relative variation of G is approximately the inverse of the present age. The predicted value of the relative variation ($G^{-1}dG/dt = 7.3 \cdot 10^{-11}$ any^{-1}) supports the observations ($G^{-1}dG/dt = (6 \pm 5) \cdot 10^{-11}$ any^{-1}).

Moreover, from the relationship between G and the energy density of the universe follows a lower limit of the measurement of the mass-energy, so we can think it is related to an underlying rest mass of the photon. That limit depends on the age of the universe, and the present is estimated to be $(1.36 \pm 0{,}01) \cdot 10^{-67}$ *kg*.

Finally it is noteworthy that the curvature of the universe and the curvature of gravitation can be described by the same geometry of hiperesferes, in according to the simple model proposed. The gravity can be understood as a local deformation of the curvature of the universe in relation to the extra spatial dimension, but in any case at cosmological curvature of the universe depends only on age, and no of gravitation, but rather the contrary: the 'gravitational constant', *G*, depends on the curvature of universe.

Appendices

A Volumetric solid angle

Taking hyperspherical coordinates, we can calculate the volume (V) corresponding to the universe (hypersurface). Therefore, we can change coordinates as:

$$\begin{cases} \mathrm{u} = T\cdot\cos\gamma \\ \mathrm{z} = T\cdot\sin\gamma\cdot\cos\alpha \\ \mathrm{x} = T\cdot\sin\gamma\cdot\sin\alpha\cdot\cos\beta \\ \mathrm{y} = T\cdot\sin\gamma\cdot\sin\alpha\cdot\sin\beta \end{cases} \tag{A.1}$$

where T is the age of the universe, like hyperspherical radius, whilst α, β and γ are the angles of a point of the hiperesfera. With the system of equations A.1, we identify the ordinary space is $r \equiv \sqrt{(x^2 + y^2 + z^2)} = T\cdot\sin\gamma$, while the spacial arc *D* at the universe is $D \equiv T\cdot\gamma$. In according to an appropriate coordinate transformation, associated with the system of equations A.1, the differential volume, *dV*, is as:

$$dV = T^3\cdot\sin^2\gamma\cdot\sin\alpha\cdot d\gamma\cdot d\alpha\cdot d\beta \tag{A.2}$$

where the domain of the triple integral goes from 0 to 2π for β, from π to 0 for α and for 0 to π for γ. With this we can calculate the volume $V(\gamma)$ closed angle γ as:

$$V(\gamma) = \iiint dV = T^3\cdot\frac{1}{2}(\gamma - \sin\gamma\cdot\cos\gamma)\Big|_0^\gamma cos\alpha\Big|_\pi^0\cdot\beta\Big|_0^{2\pi} \tag{A.3}$$

$$V(\gamma) = T^3\cdot 2\pi(\gamma - \sin\gamma\cdot\cos\gamma) \tag{A.4}$$

Furthermore, we define the local volumetric solid angle, ω_γ, according to the relationship between the volume and the cube of the distance arc *D*, as:

$$\omega_\gamma \equiv \frac{V(\gamma)}{D^3} = 2\pi(\gamma - \sin\gamma\cdot\cos\gamma)\ \gamma^{-3} \tag{A.5}$$

With such a definition, it is easy to see that its limit satisfies:

$$\lim_{\gamma\to 0}\omega_\gamma = \frac{4}{3}\pi \equiv \omega_o \tag{A.6}$$

B Time dilation and maximum velocity

According to Equation 2.6, the vector of position *s* expressed in function of time *t* and ordinary position ***r*** is:

$$s = (t,\ \ \vec{r},\ \ u_s) \tag{B.1}$$

where u_s is the component *u* of the vector *s*, which is:

$$u_s = -T\left(1 - \sqrt{1 - r^2/T^2}\right) \tag{B.2}$$

where *T* is the age of the universe. Therefore, the differential of the position is:

$$ds = (dt,\ \ \vec{v}dt,\ \ \dot{u}_s dt) \qquad \text{(B.3)}$$

where $\mathring{u}_s$ is the derivative with respect to the time of component u of the vector of position s. According to the Equation B.2, it is a function of the ordinary position r, the age of the universe T, and the ordinary velocity, v:

$$\dot{u}_s \equiv \frac{du_s}{dt} = 1 - \sqrt{1 - \frac{r^2}{T^2}} - \frac{1}{1 - \sqrt{1 - r^2/T^2}} \frac{r}{T}\left(v - \frac{r}{T}\right) \qquad \text{(B.4)}$$

One approach of $\mathring{u}_s$ is:

$$\dot{u}_s \approx \frac{r^2}{2T^2} + \left(1 + \frac{r^2}{2T^2}\right)\frac{r}{T}\left(v - \frac{r}{T}\right) \approx -\frac{r^2}{2T^2} + \frac{r}{T}v \qquad \text{(B.5)}$$

Using the Equation B.1 we can define the differential of proper time *dt'* as that interval of time measured in a system where the ordinary velocity *v* and the position *r* are zero, ie:

$$ds' = (dt',\ \ 0,\ \ 0) \qquad \text{(B.6)}$$

where *ds'* is the differential of position, measured from this reference system. Obviously, the length ds'^2 must be equal to ds^2, therefore proper time *dt'* is related to the time *dt* as:

$$dt'^2 = dt^2\left(1 - v^2 - \dot{u}_s{}^2\right) = dt^2\left(1 - w^2\right) \qquad \text{(B.7)}$$

where w is the total space velocity, ie, $w^2 = v^2 + \mathring{u}_s{}^2$. From Equation B.7 and rewriting the relationship beetwen *dt* and *dt'*, we obtain:

$$dt = \frac{dt'}{\sqrt{1 - w^2}} \qquad \text{(B.8)}$$

Therefore, an observer measure a real time *dt* if and only if $w \leq 1$, so the limit velocity of the information is w = 1, as in Einstein's special relativity. Thus, the expansion of the observable universe has a maximum angle, where the velocity is 1. In order to find the angle using the Equation B.3, we can use $r/T = \sin\gamma$ and the object is comoving with respect to universe, then the velocity of change of γ is zero, then:

$$u_s = -T\left(1 - \sqrt{1 - r^2/T^2}\right) = -T(1 - \cos\gamma) \qquad \rightarrow \qquad \dot{u}_s = -(1 - \cos\gamma) \qquad \text{(B.9)}$$

$$1 = w^2 = v^2 + \dot{u}_s{}^2 = 2\left(1 - \cos^2\gamma\right) \qquad \rightarrow \qquad \gamma = \frac{\pi}{4} \qquad \text{(B.10)}$$

were we use the Equation 2.9, i.e. $v = r/T$.

References

ABDEL-RAHMAN, A.-M.M. (1990): A critical density cosmological model with varying gravitational and cosmological "constants". *General Relativity and Gravitation*, **22**, 655-663.

ANDERSON, J.D.; KEESEY, M.S.W.; LAU, E.L.; STANDISH, E.M., JR.; NEWHALL, X.X. (1978): Tests of general relativity using astrometric and radio metric observations of the planets. *Acta Astronautica*, **5**, 43-61.

ANDERSON, J.D.; SLADE, M.A.; JURGENS, R.F.; LAU, E.L.; NEWHALL, X.X.; MYLES, E. (1991): Radar and spacecraft ranging to Mercury between 1966 and 1988. *Proceedings of the Astronomical Society of Australia*, **9**, 324.

ARBAD, A. I. (2003): The universe with bulk viscosity. *Chinese Journal of Astronomy and Astrophysics*, **3**, 113. arXiv:gr-qc/9812070v2

BENVENUTO, O. G.; ALTHAUS, L.G.; TORRES, D.F. (1999): Evolution of white dwarfs as a probe of theories of gravitation: the case of Brans-Dicke. *Monthly Notices of the Royal Astronomical Society*, **305**, 905-919.

BIESIADA, M.; MALEC, M. (2004): A new white dwarf constraint on the rate of change of the gravitational constant. *Monthly Notices of the Royal Astronomical Society*, **350**, 644-648.

BISNOVATYI-KOGAN, G.S. (2006): Checking the Variability of the Gravitational Constant with Binary Pulsars. *International Journal of Modern Physics D*, 15: 1047-1051.

BRANS, C.; DICKE, R. H. (1961): Mach's Principle and a Relativistic Theory of Gravitation. *Physical Review*, **124**, 925-935.

CAHILL, R.T.; KITTO, K. (2003): Michelson-Morley experiments revisited and the cosmic background radiation preferred frame. *Apeiron*, **10**: 104–117.
CARLIP, S. (2001): Quantum Gravity: a Progress Report. *Reports on Progress in Physics*, **64**, 885-942.
CHIN, C.-W.; STOTHERS, R. (1976): Limit on the Secular Change of the Gravitational Constant Based on Studies of Solar Evolution. *Physical Review Letters*, **36**, 833–835.
DAMOUR, T.; GUNDLACH, C. (1991): Nucleosynthesis constraints on an extended Jordan-Brans-Dicke theory. *Physical Review D (Particles and Fields)*, **43**, 3873-3877.
DEARBORN, D.S.; SCHRAMM, D. N. (1974): Limits on variation of G from clusters of galaxies. *Nature*, **247**, 441-443.
DEGL'INNOCENTI, S.; FIORENTINI, G.; RAFFELT, G.G.; RICCI, B.; WEISS, A. (1996): Time-variation of Newton's constant and the age of globular clusters. *Astronomy and Astrophysics*, **312**, 345-352.
DEMARQUE, P.; LAWRANCE, M.K.; GUENTHER, D.B.; NYDAM, D. (1994): The sun as a probe of varying G. *The Astrophysical jorunal*, **437**: 870-878.
DVALI, G.; GABADADZE, G; PORRATI, M. (2000): 4D Gravity on a Brane in 5D Minkowski Space. *Physical Letters B*, 485: 208-214.
DIRAC, P.A.M. (1938): A new basis for cosmology. *Proceedings of the Royal Society of London A*, **165**, 199-208.
DIRAC, P.A.M. (1975): Variation of G. *Nature*, **254**, 273.
EDDINGTON A. S. (1933): *The Expanding Universe*. London, Cambridge University Press. Pp. 128.
EINSTEIN, A. (1916), Die Grundlage der allgemeinen Relativitätstheorie, *Annalen der Physik*, **49**, 284-339.
FIXLER, J.B.; FOSTER, G.T.; MCGUIRK, J.M.; KASEVICH, M.A. (2007): Atom Interferometer Measurement of the Newtonian Constant of Gravity, *Science* **315**, 74–77.
GARCÍA-BERRO, E.; HERNANZ, M.; ISERN, J.; MOCHKOVITCH, R. (1995): The rate of change of the gravitational constant and the cooling of white dwarfs. *Monthly Notices of the Royal Astronomical Society*, **277**, 801-810.
GAZTAÑAGA, E.; GARCÍA-BERRO, E.;ISERN J.; BRAVO, E.; DOMÍNGUEZ, I. (2001): Bounds on the possible evolution of the gravitational constant from cosmological type-Ia supernovae. *Physical Review D*, **65**, 023506.
GREEN, M.; SCHWARZ, J.H.; WITTEN, E. (1987): *Superstring theory*. Cambridge University Press.
GUENTHER, D. B.; SILLS, K.; DEMARQUE, P.; KRAUSS, L. M. (1995): Sensitivity of solar g-modes to varying G cosmologies. *The Astrophysical Journal*, **445**, 148-151.
GUENTHER, D. B., DEMARQUE, P., & KRAUSS, L. M. (1998): Testing the Constancy of Newton's Gravitational Constant using Helioseismology**.** *Structure and Dynamics of the Interior of the Sun and Sun-like Stars SOHO 6/GONG 98 Workshop Abstract*, Boston, Massachusetts, p. 469.
HELLINGS, R.W.; ADAMS, P.J.; ANDERSON, J.D.; KEESEY, M.S.; LAU, E.L.; STANDISH, E.M.; CANUTO, V.M.; GOLDMAN, I. (1983): Experimental test of the Variability of G using Viking Lander Ranging Data, *Physical Review Letters*, **51**, 1609-1612.
HELLINGS, R.W.; ADAMS, P.J.; ANDERSON, J.D.; KEESEY, M.S.; LAU, E. L.; STANDISH, E.M.; CANUTE, V.M.; GOLDMAN, I. (1989): Experimental test of the variability of G using Viking lander ranging data. *International Journal of Theoretical Physics*, **28**, 1035-1041.
HINSHAW, G. ET AL. (WMAP collaboration) (2009): Five-Year Wilkinson Microwave Anisotropy Probe Observations: Data Processing, Sky Maps, and Basic Results. *The Astrophysical Journal Supplement*, **180**, 225–245.
HUBBLE, E.P. (1937): *The Observational Approach to Cosmology*, Oxford, Clarendon Press.
HOYLE, F. (1960): *The Nature of the Universe*. Harper, New York.
KASPI, V. M.; TAYLOR, J. H.; RYBA, M. F. (1994). High-precision timing of millisecond pulsars 3: Long-term monitoring of PSRs B1855+09 and B1937+21. *Astrophysical Journal*, **428**, 713–728.
KOMATSU, E.; ET AL. (WMAP collaboration) (2009): Five-Year Wilkinson Microwave Anisotropy Probe (WMAP) Observations: Cosmological Interpretation. *The Astrophysical Journal Supplement*, **180**, 330-376

KOWALSKI, D.R, ET AL. (THE SUPERNOVA COSMOLOGY PROJECT) (2008): Improved Cosmological Constraints from New, Old and Combined Supernova Datasets. *The Astrophysical Journal*, **686**, 749-778
KRAUSS, L.; WHITE, M. (1992): Gravitational Lensing and the Variation of G. *Astrophysics Journal*, **397**: 357
LIDDLE, A.R. (2003): *An Introduction to Modern Cosmology*. Chichester, Wiley.
MCELHINNY, M.W.; TAYLOR, S.R.; STEVENSON, D.J. (1978): Limits to the expansion of Earth, Moon, Mars and Mercury and to changes in the gravitational constant. *Nature,* **271**, 316-321.
MASSA, C. (1997): Einstein-like field equations with conserved source and decreasing $\wedge$ term; their cosmological consequences. *Astrophysics and Space Science*, **246**, 153-158
MOHR, P. J.; BARRY N. TAYLOR, B. N. (2005), CODATA recommended values of the fundamental physical constants: 2002, *Reviews of Modern Physics*, **77**, 1–107,
MØLLER, C. (1952): *The Theory of Relativity,* Oxford University Press, Oxford, England.
MORRISON, L.V. (1973): Rotation of the Earth from AD 1663-1972 and the Constancy of G. *Nature,* **241**, 519-520.
MÜLLER, J.; SCHNEIDER, M.; SOFFEL, M.; RUDER H. (1991): Testing Einstein's theory of gravity by analyzing lunar ranging data. *Astrophysics Journal Letters*, **382**, p. L101.
MÜLLER, J.; BISKUPEK, L. (2007): Variations of the gravitational constant from lunar laser ranging data. *Classical and Quantum Gravity*, **24**, 4533.
MÚNERA, H.A. (2009): Towards the reinstatement of absolute space, and some possible cosmological implications. *The ICFAI University Journal of Physics*, 2: 7 - 24.
OLIVE, K.A.; POSPELOV, M.; QIAN, Y.-Z.; COC, A.; CASSÉ, M.; VANGIONI-FLAM, E. (2002): Constraints on the variations of the fundamental couplings. *Physical Review D,* **66**: 045022.
O'NEILL, B. (1983): *Semi-Riemannian Geometry: With Applications to Relativity*. Ed. Academic Press. 468 p. eISBN: 978-0-08-057057-0.
OVERDUIN, J. M.; WESSON, P. S. (1997): Kaluza-Klein Gravity. *Physics Reports,* **283**, 303-378.
REASENBERG, R.D.; SHAPIRO, I.I. (1978): *On the Measurement of Cosmological Variations of the Gravitational Constant*, in HALPERN, L. (Ed.), Gainesville, University Presses of Florida: 21.
RIESS, A.G.; FILIPPENKO, A.V.; CHALLIS, P.; CLOCCHIATTI, A.; DIERCKS, A.; GARNAVICH, P.M.; GILLILAND, R.L.; HOGAN, C.J.; JHA, S.; K., ROBERT P.; LEIBUNDGUT, B.; PHILLIPS, M.M.; REISS, D.; SCHMIDT, B.P.; SCHOMMER, R.A.; S.R. CHRIS; SPYROMILIO, J.; STUBBS, C.; SUNTZEFF, N.B.; TONRY, J. (1998): Observational Evidence from Supernovae for an Accelerating Universe and a Cosmological Constant. *The Astronomical Journal,* **116**, 1009-1038.
ROBERTSON, H.P. (1929): The Uncertainty Principle. *Physical Review,* **34**, 163–164.
ROVELLI, C. (2004): *Quantum Gravity*. Cambridge University Press.
SALAM, A.; WIGNER, E. (1972): *The Fundamental Constants and their Time Variation*. Cambridge University Press, Cambridge.
SALVATORE, P.; LONGONI, R. (2005): "Configuration spaces are not homotopy invariant", *Topology,* 44: 375–380.
SCHWARZSCHILD, K. (1916a): Uber das Gravitationsfeld eines Massenpunktes nach der Einsteinschen Theorie, *Sitzungsberichte der Deutschen Akademie der Wissenschaften zu Berlin, Klasse fur Mathematik, Physik, und Technik,* pp 189.
SCHWARZSCHILD, K. (1916b): Uber das Gravitationsfeld einer Kugel aus inkompressibler Flussigkeit nach der Einsteinschen Theorie, *Sitzungsberichte der Deutschen Akademie der Wissenschaften zu Berlin, Klasse fur Mathematik, Physik, und Technik*, pp 424.
SHAPIRO, I.I.; ASH, M.E.; INGALLS, R.P.; SMITH, W.B.; CAMPBELL, D.B.; DYCE, R.B., JURGENS, R.F., PETTENGILL G. H.(1971): Fourth Test of General Relativity: New Radar Result. *Physical Review Letters,* **26**, 1132–1135
SISTERNA, P.D.; VUCETICH, H. (1994): Cosmology, oscillating physics, and oscillating biology. *Physical Review Letters,* **72**, 454–457.
SISTERNA, P.D.; VUCETICH, H. (1991): Time variation of fundamental constants. II. Quark masses as time-dependent parameters. *Physical Review D* **44**, 3096–3108.
SMOOT, G. F.; BENNETT, C. L.; KOGUT, A.; WRIGHT, E. L.; AYMON, J.; BOGGESS, N. W.; CHENG, E. S.; DE AMICI, G.; GULKIS, S.; HAUSER, M. G.; HINSHAW, G.; JACKSON, P. D.; JANSSEN, M.; KAITA, E.; KELSALL, T.; KEEGSTRA, P.; LINEWEAVER, C.;

LOEWENSTEIN, K.; LUBIN, P.; MATHER, J.; MEYER, S. S.; MOSELEY, S. H.; MURDOCK, T.; ROKKE, L.; SILVERBERG, R. F.; TENORIO, L.; WEISS, R.; WILKINSON, D. T. (1992): Structure in the COBE differential microwave radiometer first-year maps. Astrophysical Journal, Part 2: L1-L5.

SPERGEL, D. N.; ET AL. (WMAP collaboration) (2007). Three-Year Wilkinson Microwave Anisotropy Probe (WMAP) Observations: Implications for Cosmology. *Astrophysical Journal Supplement* **170**, 377.

SPERGEL, D. N.; ET AL. (WMAP collaboration) (2003): First year Wilkinson Microwave Anisotropy Probe (WMAP) observations: determination of cosmological parameters, *Astrophysical Journal Supplement* **148**, 175.

SZABÓ, G.M.; GERGELY, L.A.; KERESZTES, Z. (2007): The luminosity-redshift relation in brane-worls: II. Confrontation with experimental data. *PMC Physics A* **1**: 8.

TEGMARK, M.; ET AL. (WMAP collaboration) (2004): Cosmological Parameters from SDSS and WMAP. *Physical Review D,* **69,** 103501.

THORSETT, S. E. (1996): The Gravitational Constant, the Chandrasekhar Limit, and Neutron Star Masses. *Physical Review Letters,* **77**,1432–1435.

VAN FLANDERN, T.C. (1981): Is the Gravitational Constant Changing. *Astrophysical Journal*, **248**, 813-816.

WANG, J. (1991): Astrophysical constraints on the gravitational constant. *Astrophysics and Space Science,* **184**, 31-36.

WALD, R. M. (1984): *General Relativity,* Chicago University Press.

WEINBERG, S. (1972): *Gravitation and Cosmology: principles and applications of the general theory of relativity*. Wiley.

WEINBERG, S. (1995): *The Quantum Theory of Fields I: Foundations*, Cambridge University Press.

WEINBERG, S. (1996): *The Quantum Theory of Fields II: Modern Applications*, Cambridge University Press.

WEINBERG, S. (2000): *The Quantum Theory of Fields III: Supersymmetry*, Cambridge University Press.

WILLIAMS, J.G.; SINCLAIR, W.S.; YODER, C.F. (1978): Tidal acceleration of the Moon. *Geophysical Research Letters*, **5**, 943-946.

WILLIAMS, J.G.;NEWHALL, X.X; DICKEY, J.O. (1996): Relativity parameters determined from lunar laser ranging. *Physical Review D*, **53**, 6730–6739.

WU, Y.; WANG, Z. (1986): Time variation of Newton's gravitational constant in superstring theories. *Physical Review Letters,* **57**, 1978-1981.

WUENSCH, D. (2003): The 5th Dimension: Theodor Kaluza's Groundbreaking Idea. *Annalen der Physik*, **9**, 519-542.